SLP NO.01

Interactive gaming in spatial networks

---The multi-agent modeling (simulation) approach

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Key ideas

In complex social environment entangled with complex social relationships, people need to be smart and good at mentalizing others' minds to make ideal social behaviors (decisions) to get more benefits.
But on the other hand, human are "cognitive misers", too much mentalizing process, such as Theory of Mind (ToM) may consume energy and further be harmful to the social agents.

Old Chinese saying:

- 🍊 Fortune favours fools（傻人有傻福）
- 🍊 A man of great wisdom behaves like a fool（大智若愚）

Key ideas

In this project, I want to simulate an interactive computational game in large-scale spatial networks to investigate following questions:

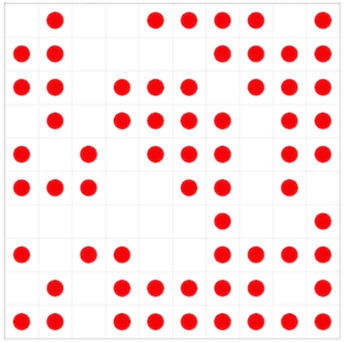
(1) From evolutional perspective, would the high-level ToM agents (very good at predicting others' minds in decision making) survive better than lower-level ToM agents?

(2) How would the ratio of different ToM agents (in current simulation, I set 0-ToM, 1-ToM, 2-ToM, 3-ToM in all) influence the final survived rate?

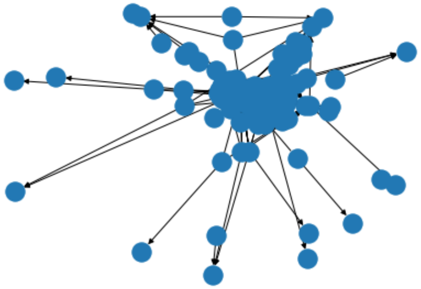
(3) How would the relationship between *cognitive-cost* and *game-earned* would influence the final survived rate?

Spatial Networks

- Agents meet each other in a grid network, and interacts with its Moore neighbors.



- scale-free (social) network



Interactive game

Penny competitive game

zero-sum game

If both pennies show heads or both show tails, Bob pays Adam $1

If one penny shows heads and the other shows tails, Adam pays Bob $1

Adam / Bob	Heads	Tails
Heads	(a) +1, -1	(b) -1, +1
Tails	(c) -1, +1	(d) +1, -1

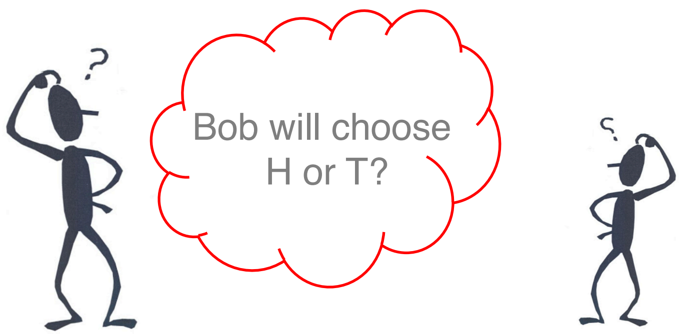
Ideal strategy for Adam:
Always try to keep **same** penny with Bob’s

Ideal strategy for Bob:
Always try to keep **different** penny with Adam’s

Interactive computational modeling

Inferring other minds (mentalizing) model

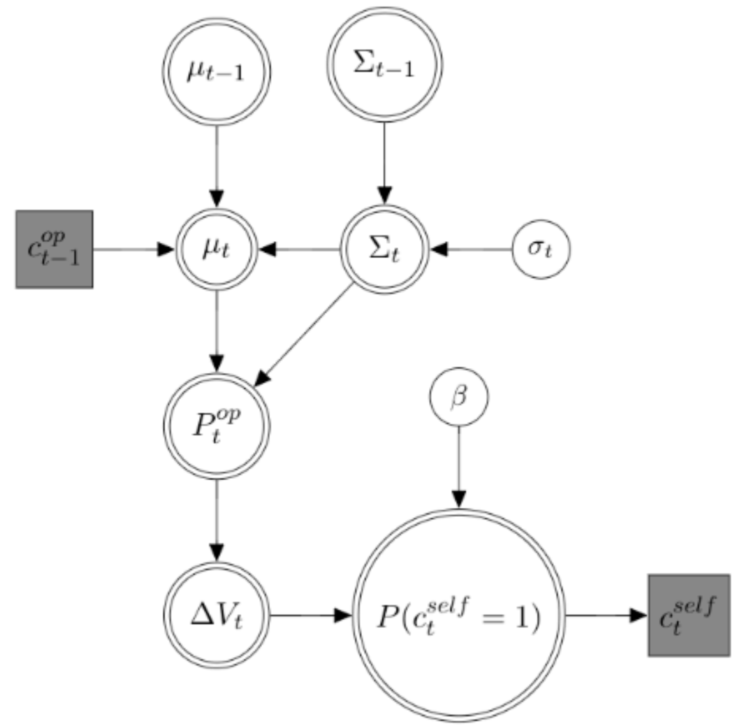
0-ToM



Adam

Bob

Graphic model for 0-ToM



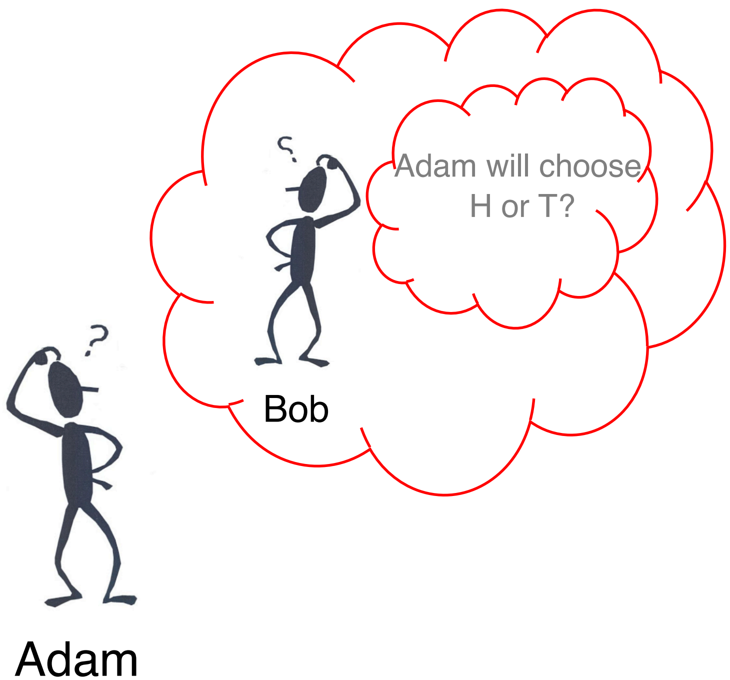
Shaded variables are observed (choice), squares are discrete while circles are continuous, and double bordered variables are deterministic and unobserved.

Examples are based on Adam’s thinking

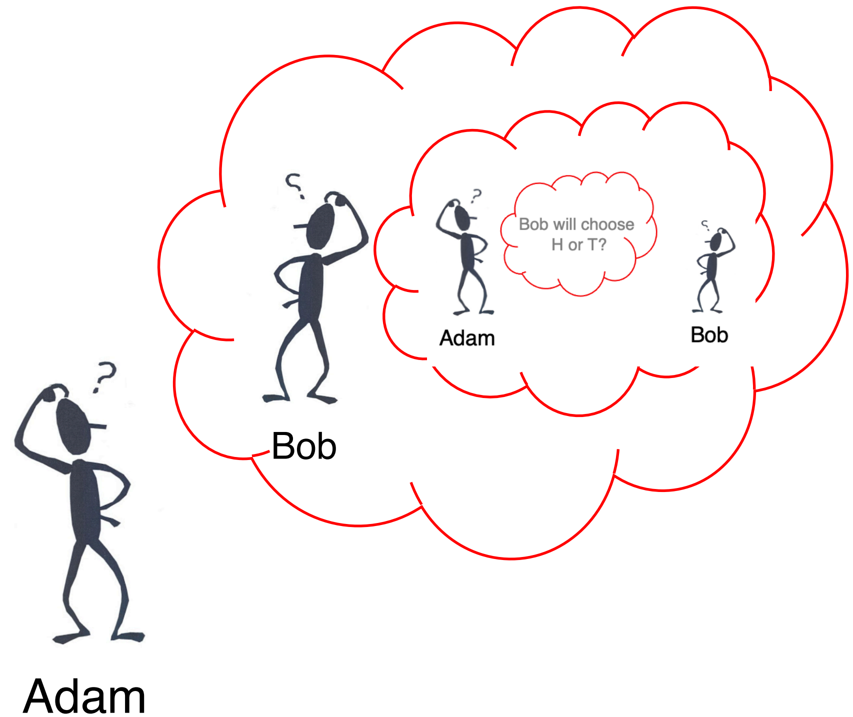
Interactive computational modeling

Inferring other minds (mentalizing) model

1-ToM



2-ToM

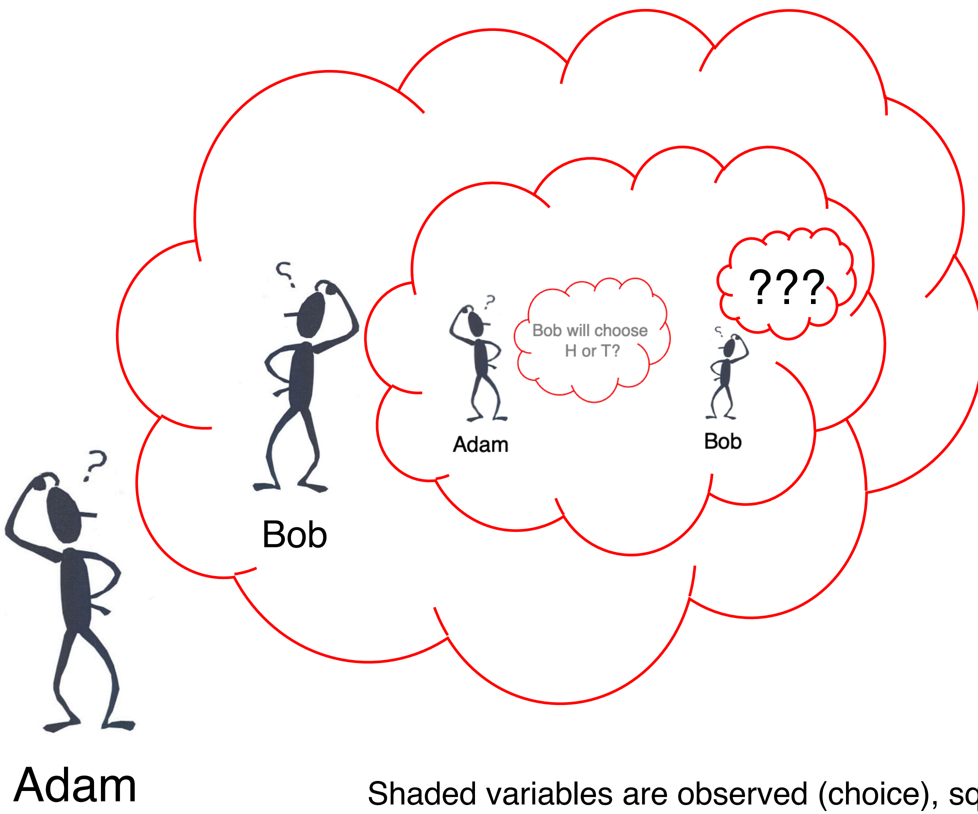


Examples are based on Adam’s thinking

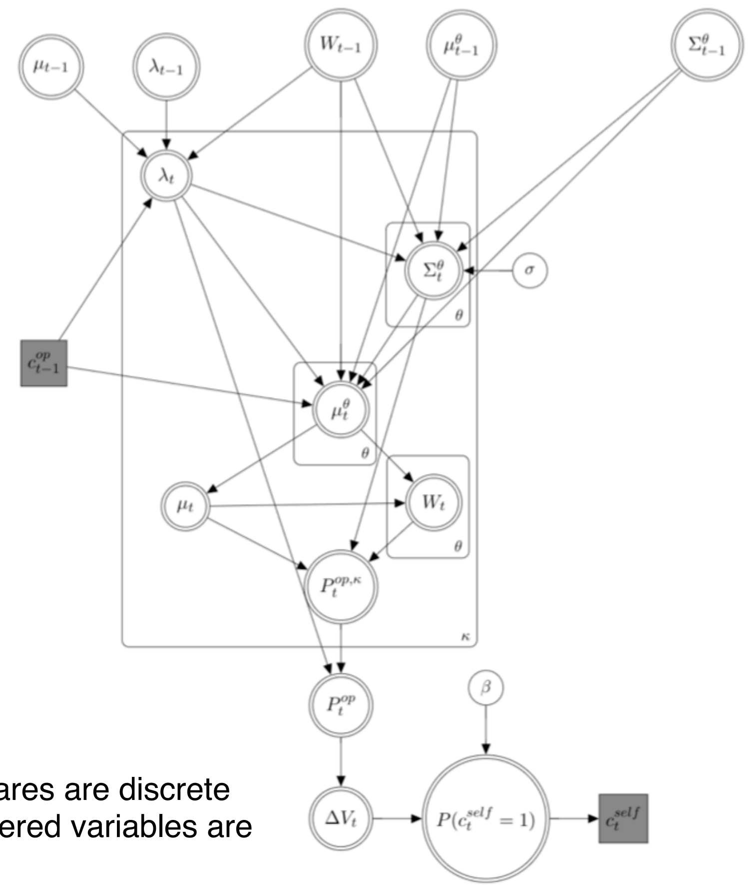
Interactive computational modeling

Inferring other minds (mentalizing) model

k-ToM (k > 1)



Graphic model for k-ToM



Shaded variables are observed (choice), squares are discrete while circles are continuous, and double bordered variables are deterministic and unobserved.

Examples are based on Adam's thinking

Interactive computational modeling

0-ToM computational process

🍟 Agents estimate their opponents' parameter θ to learn the choice probability of their opponents P_t^{op}

🍟 Since 0-ToM assumes its opponent will use a random phenoma, so let choice probability parameter is estimated as a normal distribution with mean μ and variance Σ, both of these two parameters are updated trial by trial

🍟 Variance Σ is updated in this way:

s *is the sigmoid function*, $t-1$ *(last trial)*

$$\Sigma_t^0 \approx \frac{1}{\frac{1}{\Sigma_{t-1}^0+\sigma^0}+s(\mu_{t-1}^0)(1-s(\mu_{t-1}^0))}$$

🍟 Mean μ is updated in this way:

$$\mu_t^0 \approx \mu_{t-1}^0 + \Sigma_t^0(c_{t-1}^{op} - s(\mu_{t-1}^0))$$

🍟 Probability of opponent choosing 1:

$$p_t^{op} \approx s\left(\frac{\mu_t^0}{1+(\Sigma_t^0+\sigma^0)3/\pi^2)}\right)$$

Interactive computational modeling

k-ToM computational process

Things will be much more complex in k-ToM situation:

🍟 First, calculating the $\lambda_t^{k,kappa}$ which denotes k-ToM's estimated prob at trial t of its opponent having the sophistication level *kappa:*

$$\lambda_t^{k,kappa} \approx \left(\frac{\lambda_{t-1}^{k,kappa} P_{t-1}^{op,kappa}}{\Sigma_{kappa'<k}\, \lambda_{t-1}^{k,kappa'} P_{t-1}^{op,kappa'}}\right)^{c_{t-1}^{op}} \left(\frac{\lambda_{t-1}^{k,kappa} P_{t-1}^{op,kappa}}{\Sigma_{kappa'<k}\, \lambda_{t-1}^{k,kappa'} P_{t-1}^{op,kappa'}}\right)^{1-c_{t-1}^{op}}$$

🍟 Introducing the parameter $W_{t-1}^{k,kappa,\theta}$, the gradient from last trial of the relation between each parameter estimate μ^{θ} and the choice prob estimate μ, for each level kappa (ToM level of opponents)

$$W_{t-1}^{k,kappa,\theta} = \frac{d\mu^{k,kappa}}{d\mu^{k,kappa,\theta}}$$

Interactive computational modeling

κ-ToM computational process

🍟 Variance Σ is updated in this way:

$$\Sigma_{t-1}^{k,kappa} = (\Sigma_{t-1}^{k,kappa,\theta})^{T} (W_{t-1}^{k,kappa,\theta})^{2})$$

$$\Sigma_{t}^{k,kappa,\theta} \approx \frac{1}{\frac{1}{\Sigma_{t-1}^{k,kappa,\theta}+\sigma^{k}} + s(\mu_{t-1}^{k,kappa,\theta})(1-s(\mu_{t-1}^{k,kappa,\theta}))\lambda_{t}^{k,kappa}(W_{t-1}^{k,kappa,\theta})^{2}}$$

🍟 Mean μ is updated in this way:

$$\mu_{t}^{k,kappa,\theta} \approx \mu_{t-1}^{k,kappa,\theta} + W_{t-1}^{k,kappa,\theta} \Sigma_{t}^{k,kappa,\theta} \lambda_{t}^{k,kappa} (c_{t-1}^{op} - s(\mu_{t-1}^{k,kappa,\theta}))$$

🍟 Probability of opponent choosing 1:

$$p_{t}^{op,kappa} \approx s\left(\frac{\mu_{t}^{k,kappa}}{1+\left(\Sigma_{t}^{k,kappa} + \sigma^{k,kappa}\right)3/\pi^{2})}\right)$$

Agent-based modeling settings

- simulating 100 agents, in 10000 generations

- simulating 4 kinds of ToM-agent:

0-ToM; 1-ToM; 2-ToM; 3-ToM (I did not set 4-ToM agents, details see *Devaine et al., 2014, PCB*)

with key modeling parameters:

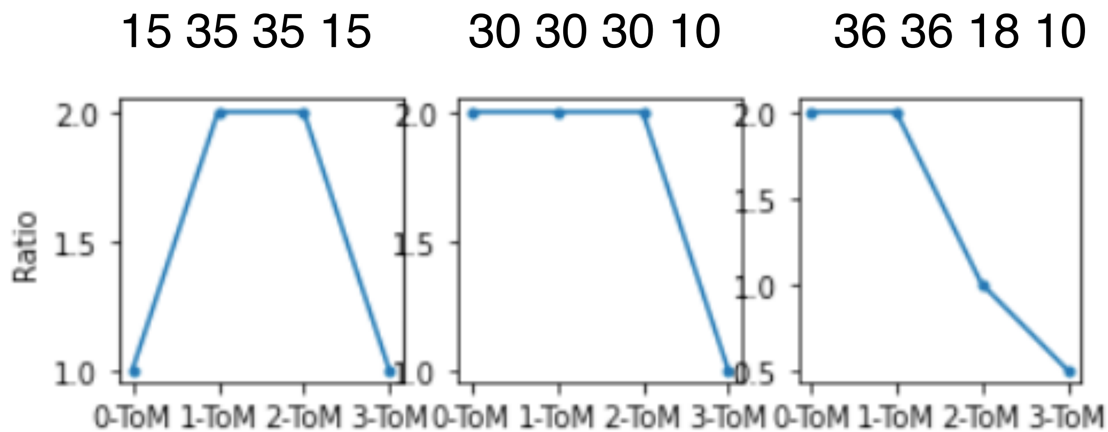
['volatility': -2, ’ temperature': -4, 'dilution': 0.4, 'bias': 0.5]

- assuming all agents will keep his ToM-level unchanged during whole life

- Higher order ToM will also show higher energy cost (energy cost ratio see later slide)

Agent-based modeling settings

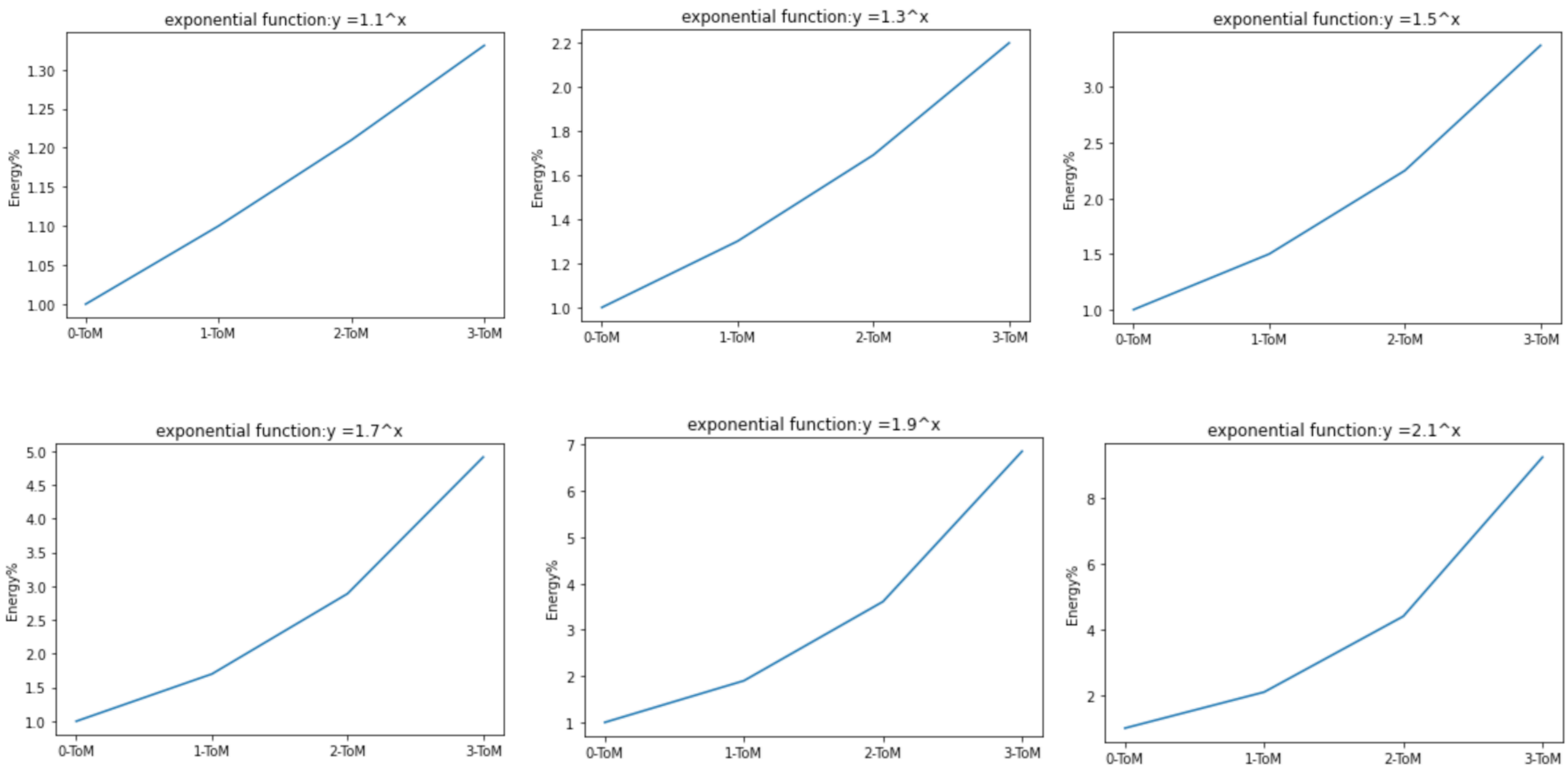
🦴 ratio among all kinds of k-ToM agents:



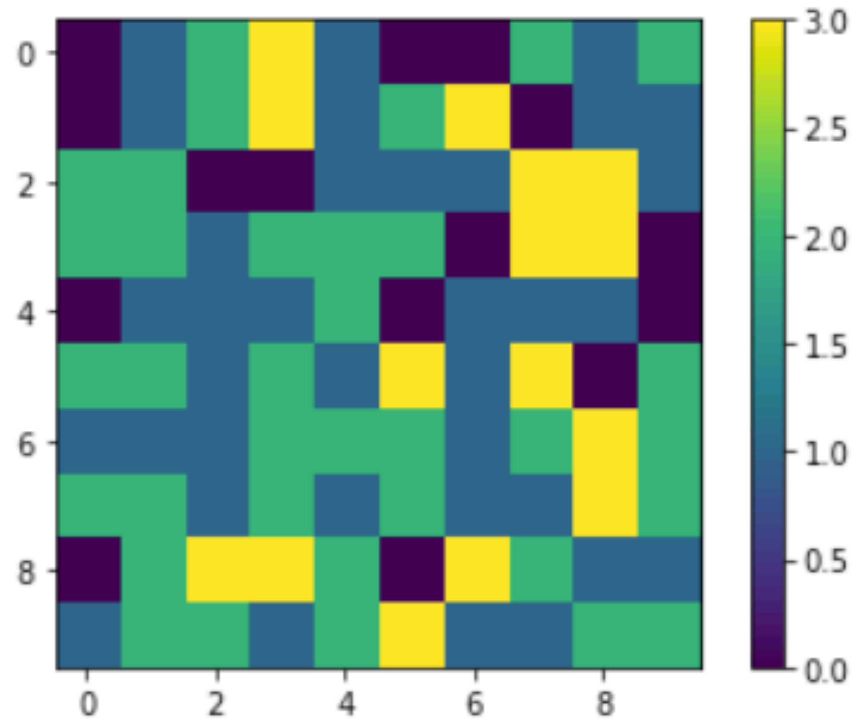
Agent-based modeling settings

- Relationship between k level of mentalizing and energy consuming

Y (energy cost%) = a^{x} (a∈ [1.1,1.3,1.5,1.7,1.9,2.1])



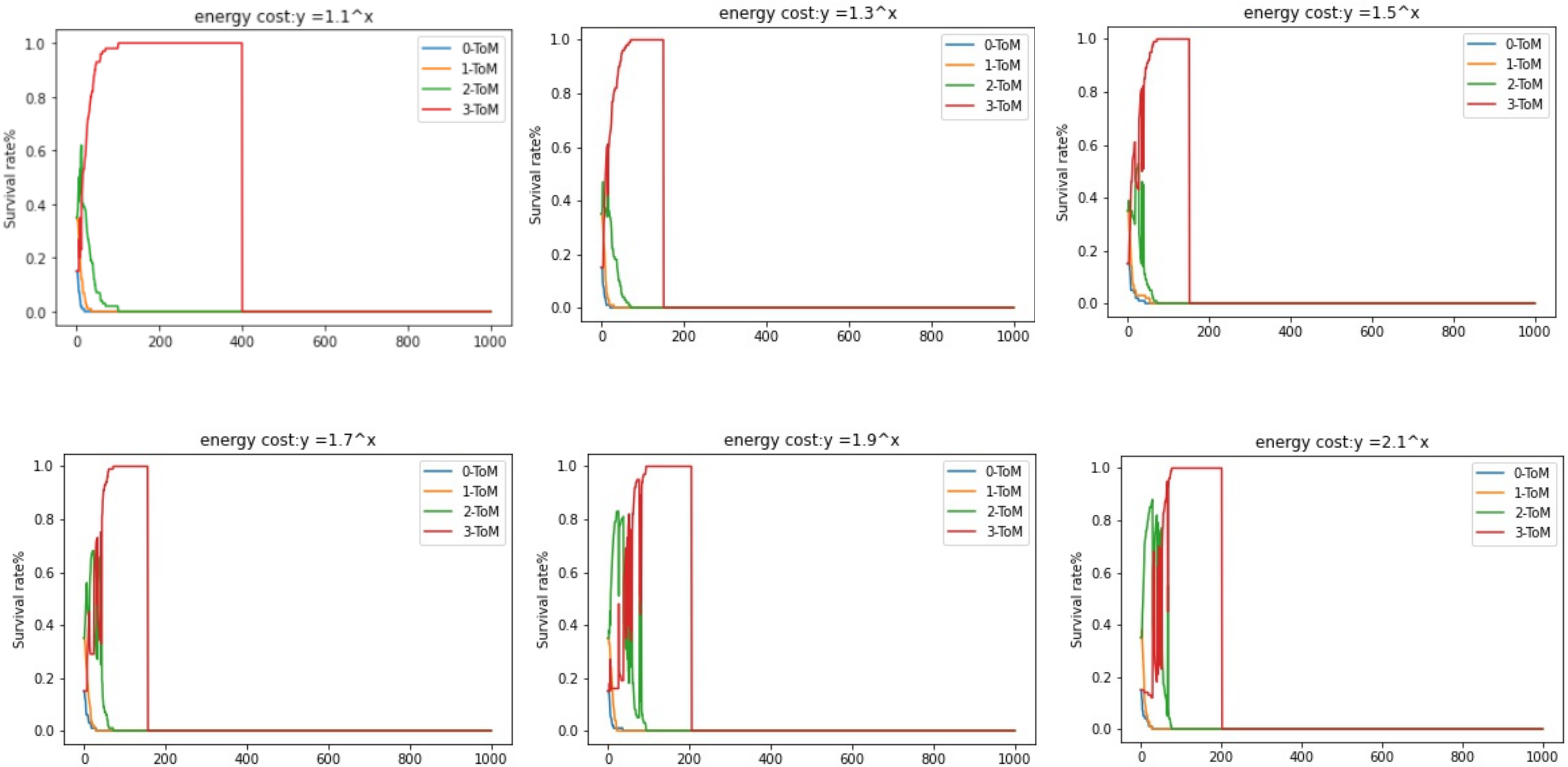
Agent-based modeling settings



0 = 0-TOM
1 = 1-TOM
2 = 2-TOM
3 = 3-TOM

Results for grid network

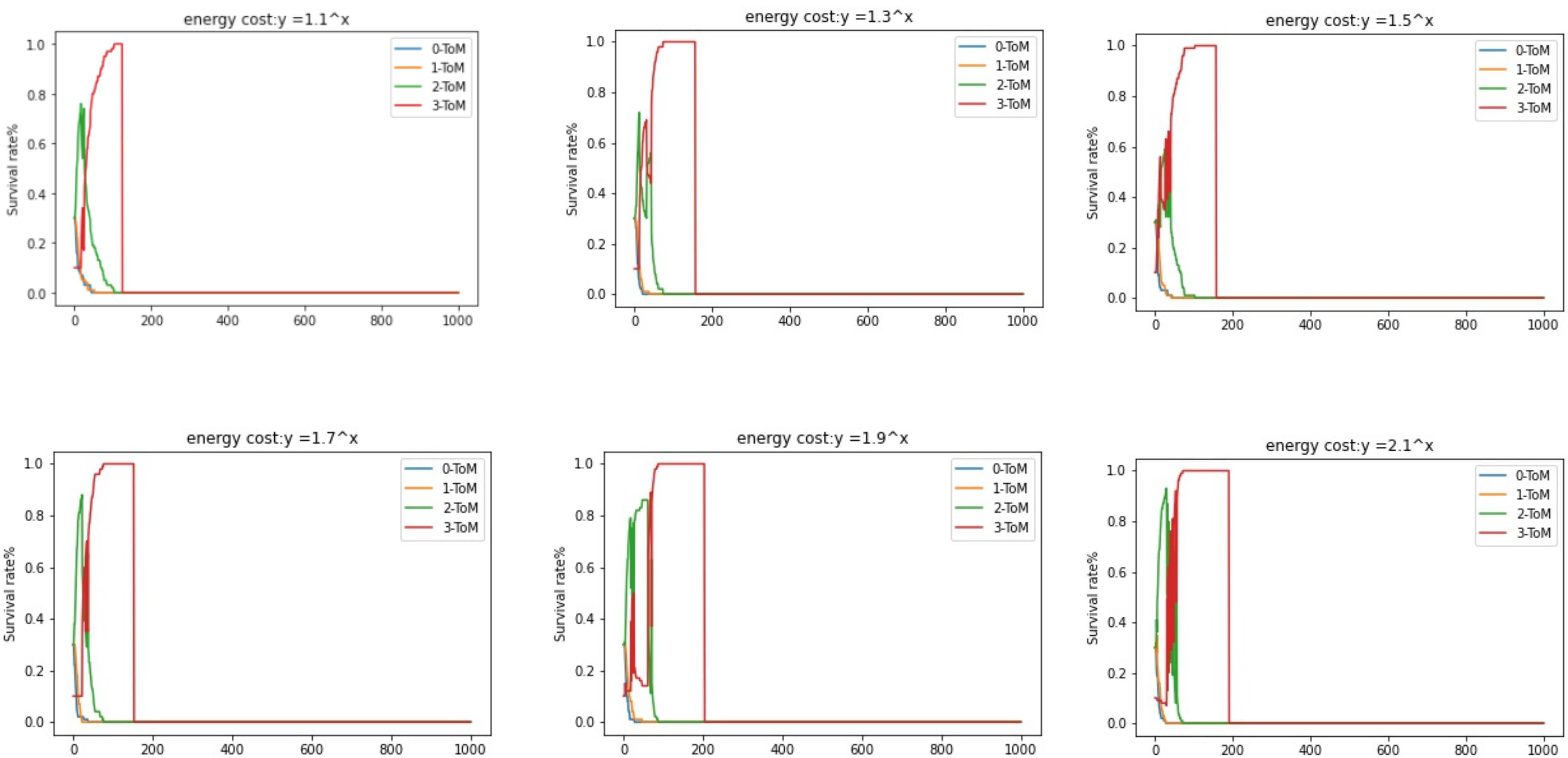
Ratio1 {0-tom=15; 1-tom=35; 2-tom=35;3-tom=15}



In any energy cost ratio, ToM-3 agents win, then comes the ToM-2 agents

Results for grid network

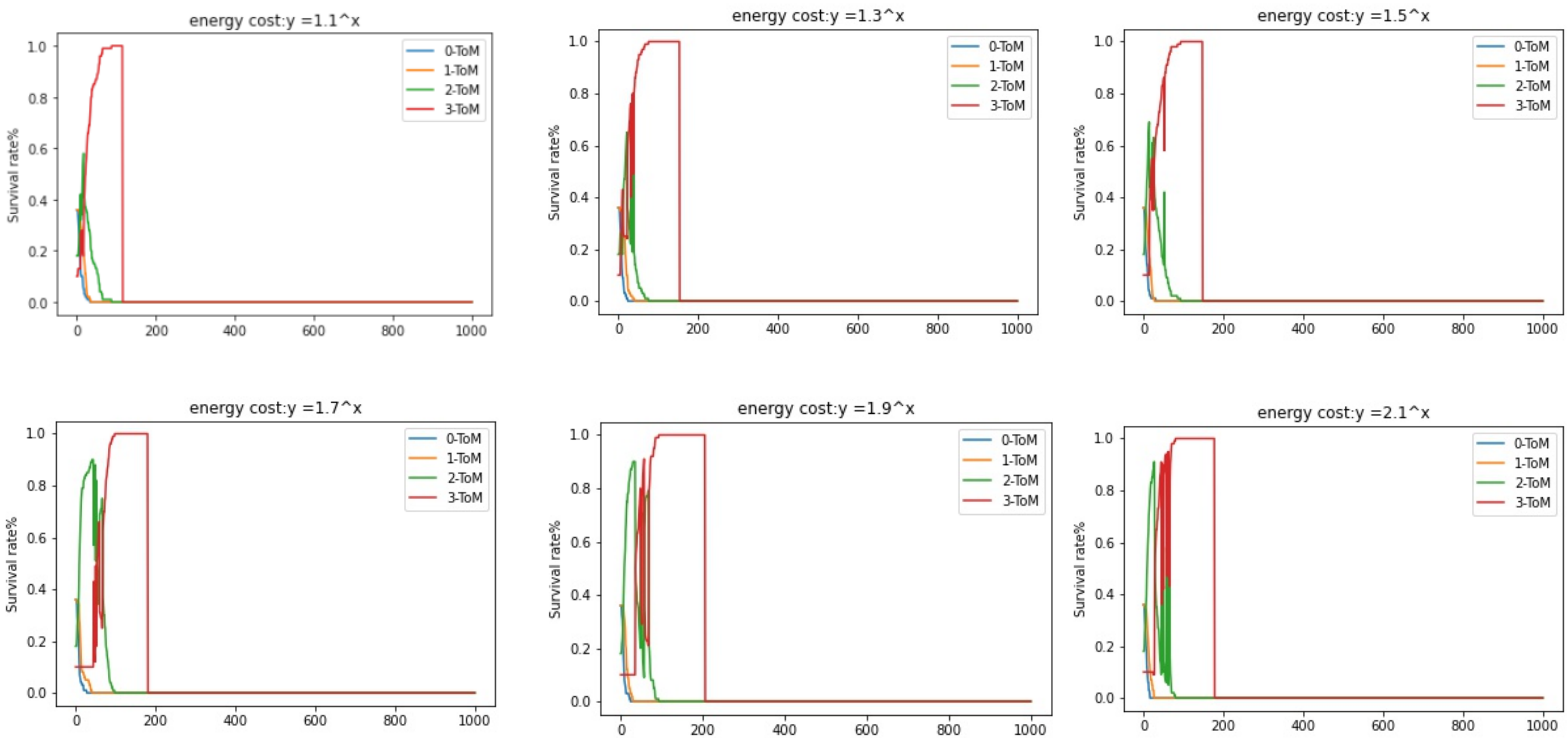
Ratio2 {0-tom=30; 1-tom=30; 2-tom=30;3-tom=10}



In any energy cost ratio, ToM-3 agents win, then comes the ToM-2 agents

Results for grid network

Ratio3 {0-tom=36; 1-tom=36; 2-tom=18;3-tom=10}



In any energy cost ratio, ToM-3 agents win, then comes the ToM-2 agents

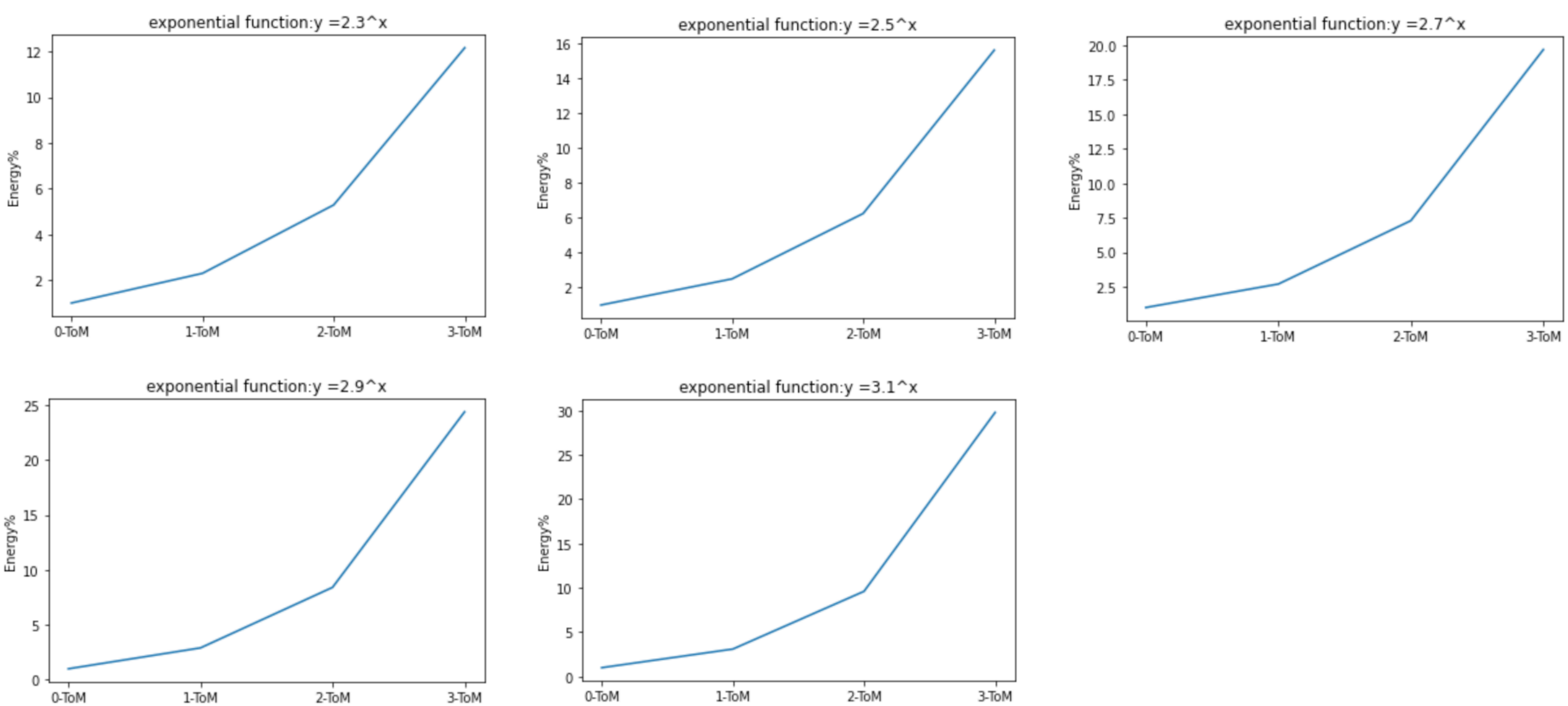
Further question --- Finding the Turn Point

In which energy threshold would lead high-order ToM agent disappear?

Adding 5 kinds of energy cost ratio

Y (energy cost%) = a^x (a∈ $[2.3,\ 2.5,\ 2.7,\ 2.9,\ 3.1]$)

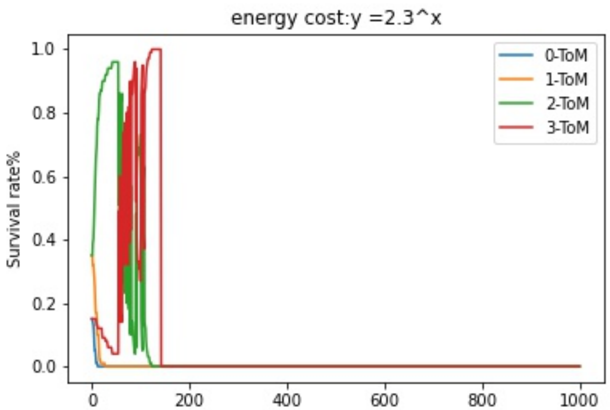
Let us wait what thing will happen under these energy cost ratios?

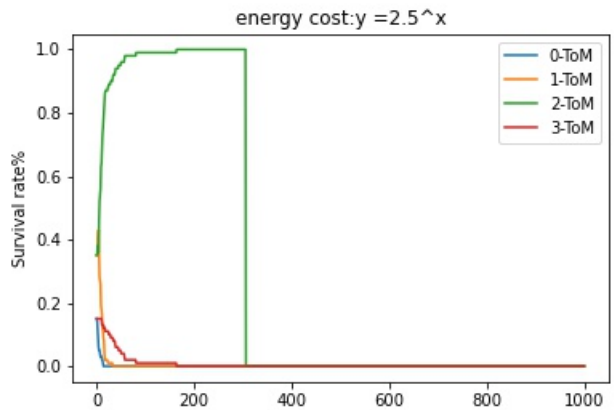


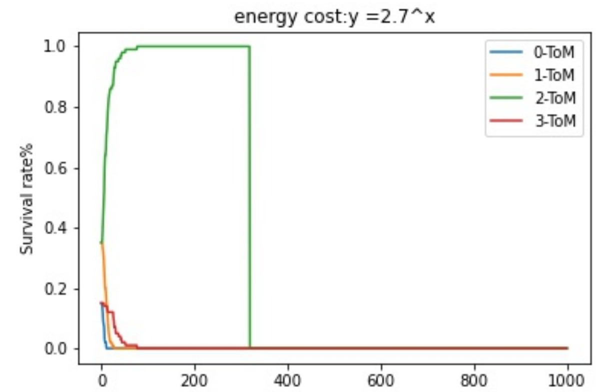
Results for grid network

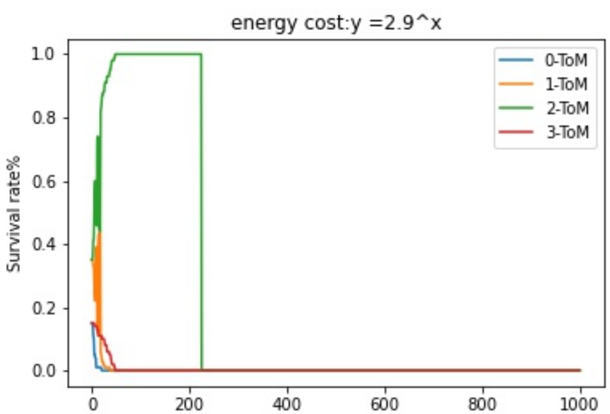
Y (energy cost%) = a^x (a∈ $[2.3,\ 2.5,\ 2.7,\ 2.9,\ 3.1]$)

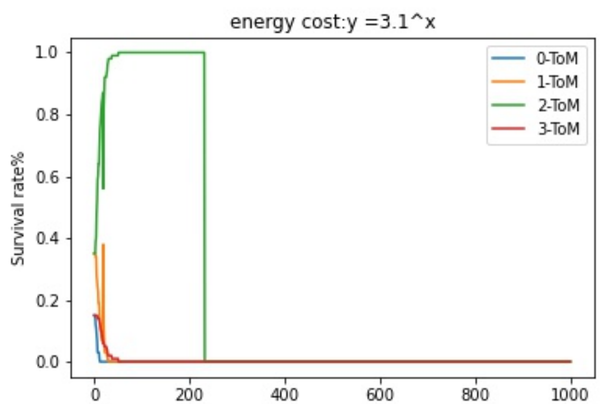
Ratio1 {0-tom=15; 1-tom=35; 2-tom=35; 3-tom=15}









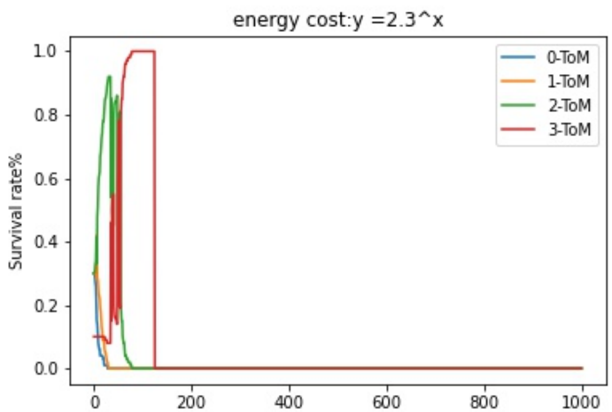


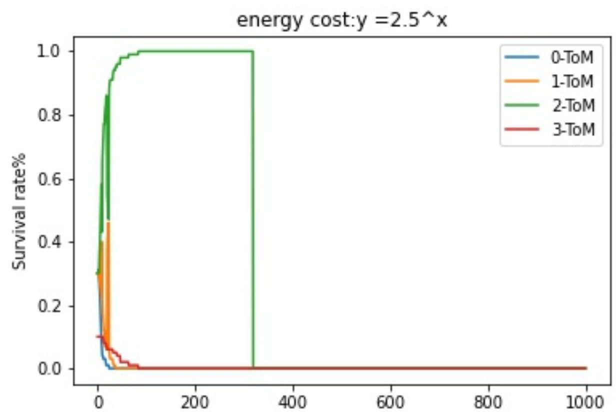
Stable turn point (ToM-2 wins!) is y = 2.5^x

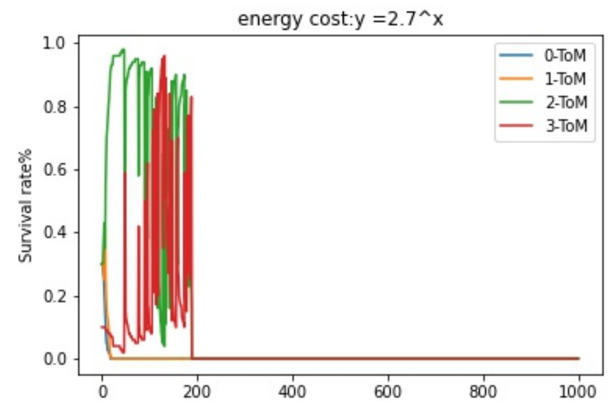
Results for grid network

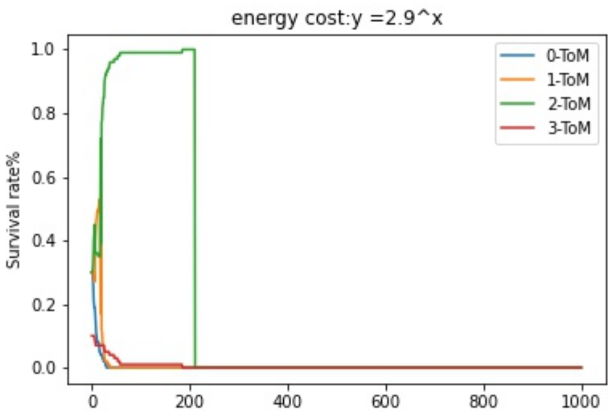
Y (energy cost%) = a^x (a∈ $[2.3,\ 2.5,\ 2.7,\ 2.9,\ 3.1]$)

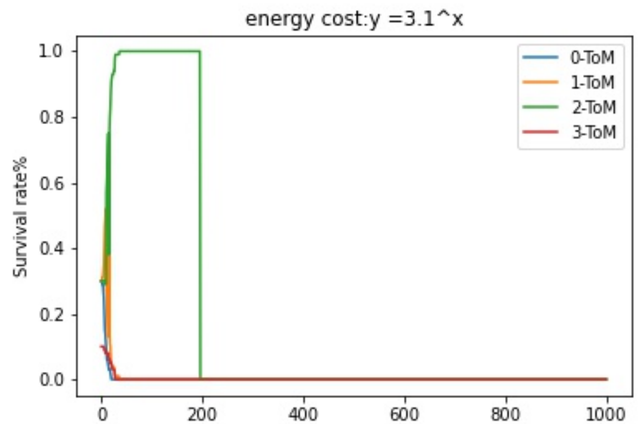
Ratio2 {0-tom=30; 1-tom=30; 2-tom=30; 3-tom=10}









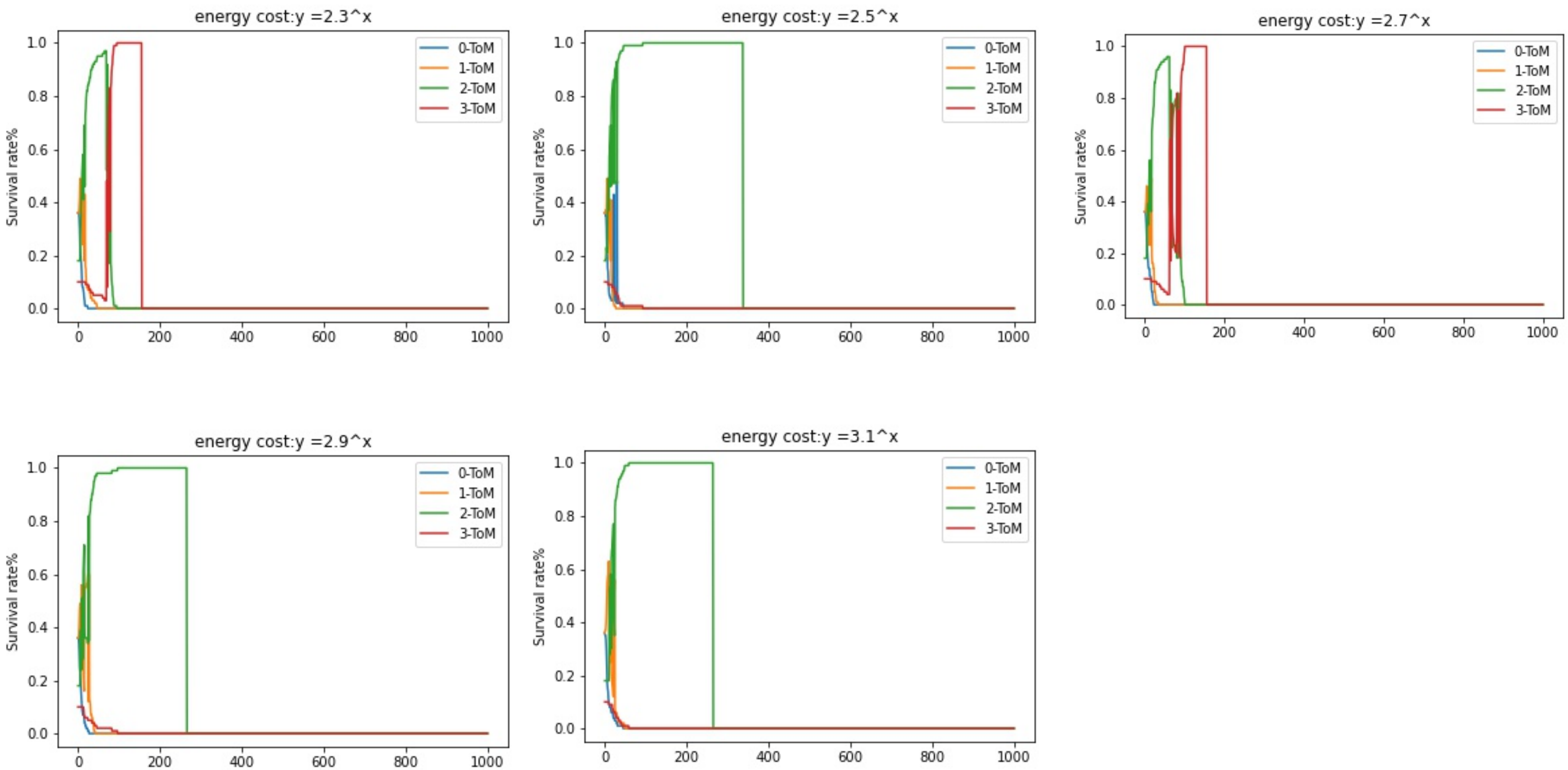


Stable turn point (ToM-2 wins!) is y = 2.9^x

Results for grid network

Y (energy cost%) = a^x (a∈ $[2.3,\ 2.5,\ 2.7,\ 2.9,\ 3.1]$)

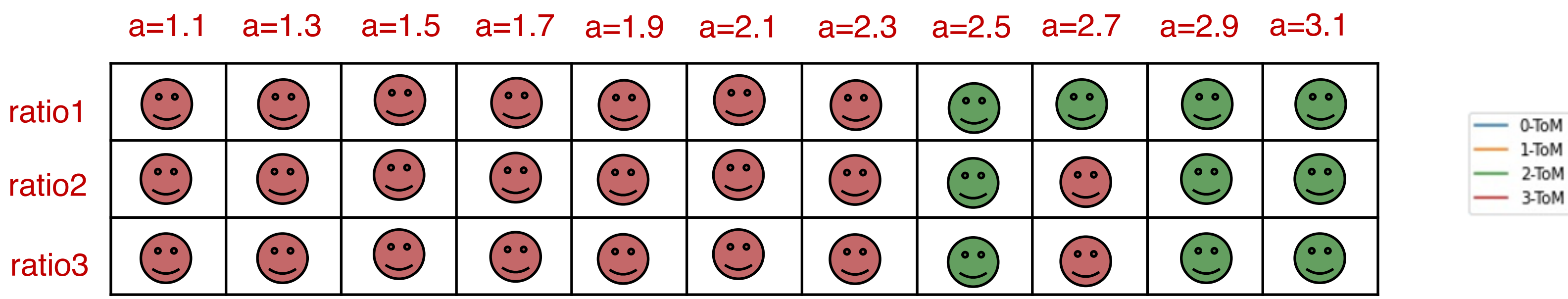
Ratio3 {0-tom=36; 1-tom=36; 2-tom=18; 3-tom=10}



Stable turn point (ToM-2 wins!) is y = 2.9^x

Results for grid network

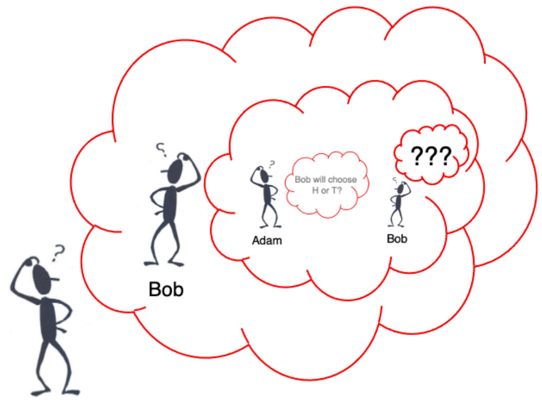
Y (energy cost%) = a^x

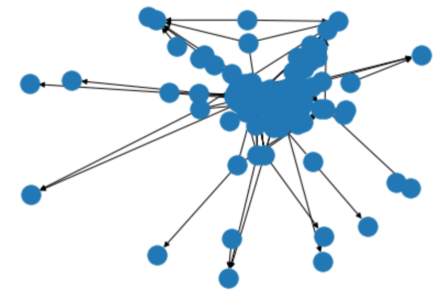


In random grid network (space), ToM-3 agents have obvious advantages to get benefits, *if and only if (iif)* in the condition when energy cost ratio is less than y = 2.9^x

Inferring other minds (mentalizing) model

k-ToM (k > 1)



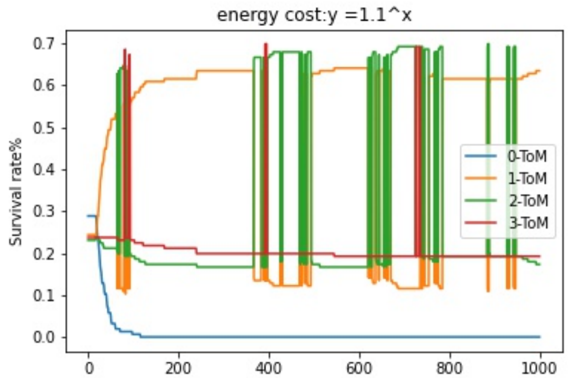


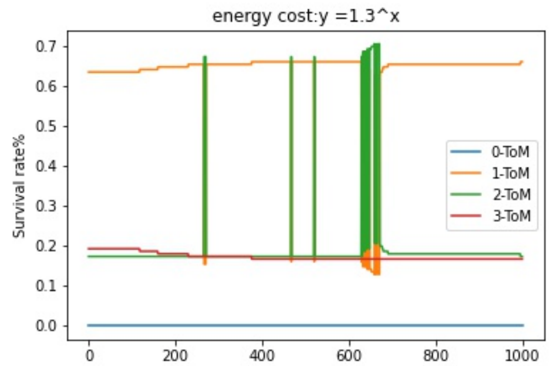
Part II Results for social network

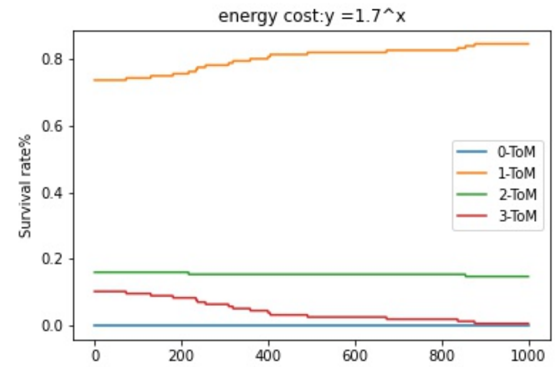
Which k-ToM agents would be more suitable to be social hub?

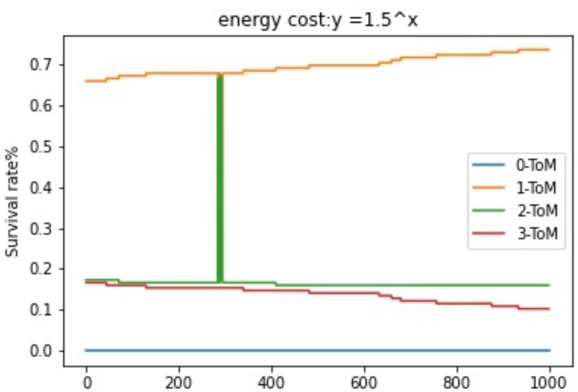
ToM-0 is social hub

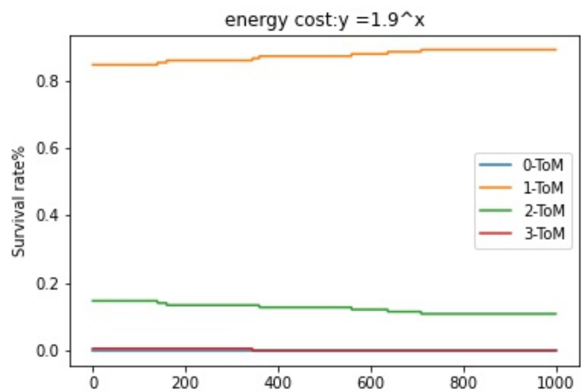
Results for social network

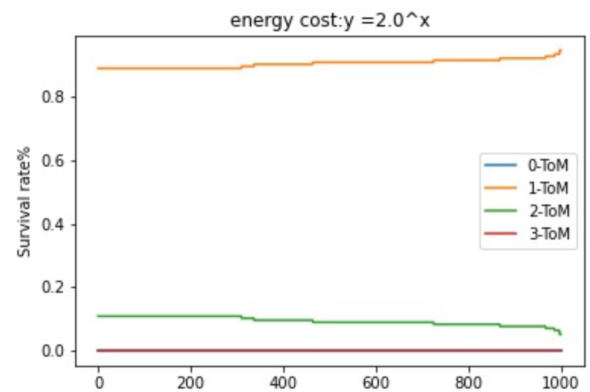


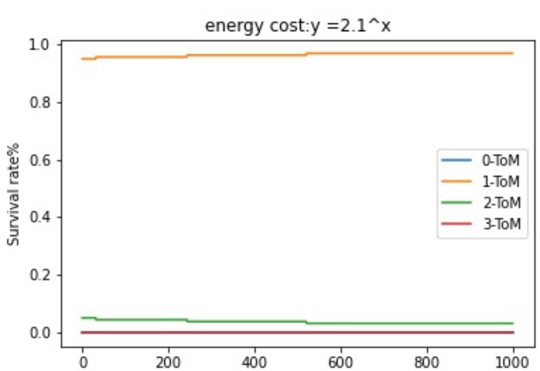




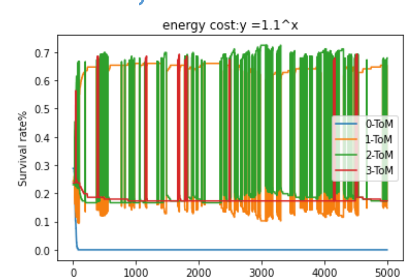






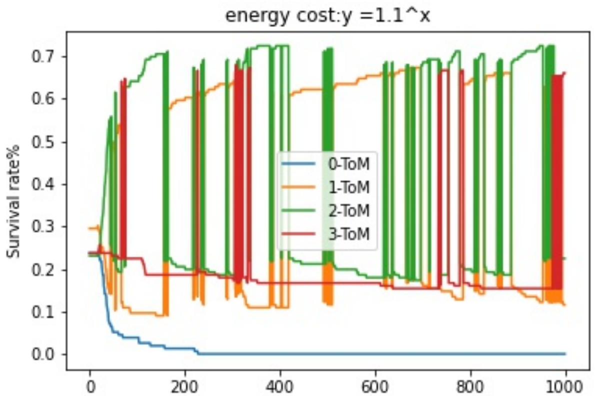


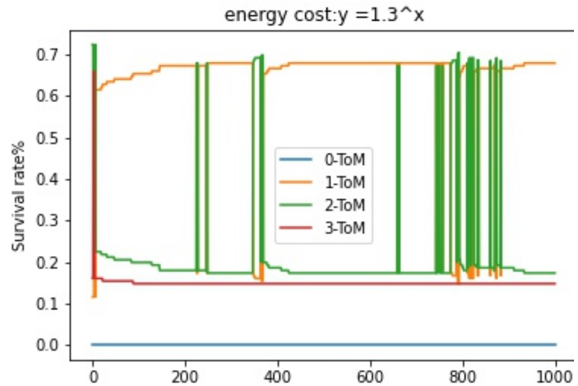
ToM-1 wins! But it seems that in Y (energy cost%) = 1.1^{x} the system was not stable, even when I added the episode into 5,000

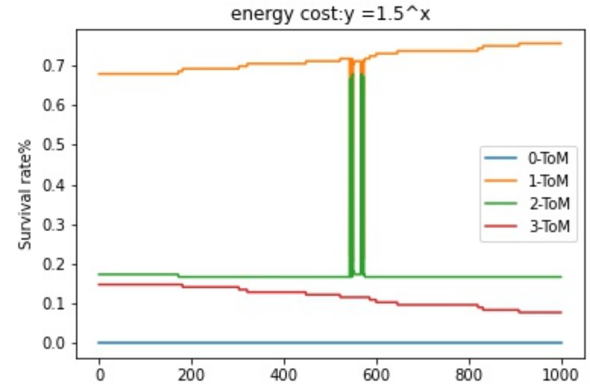


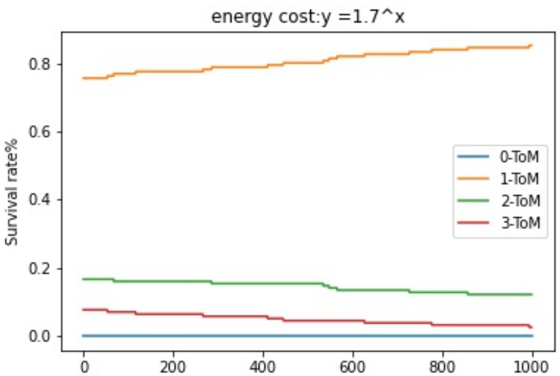
ToM-1 is social hub

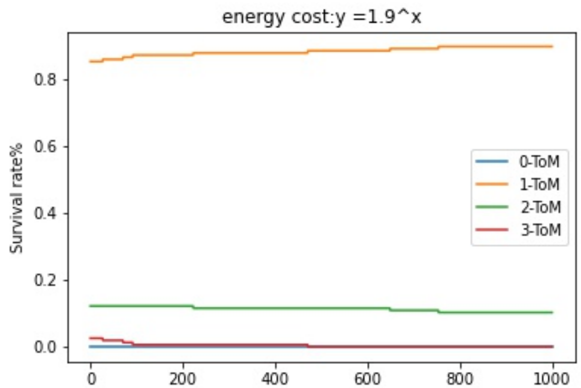
Results for social network

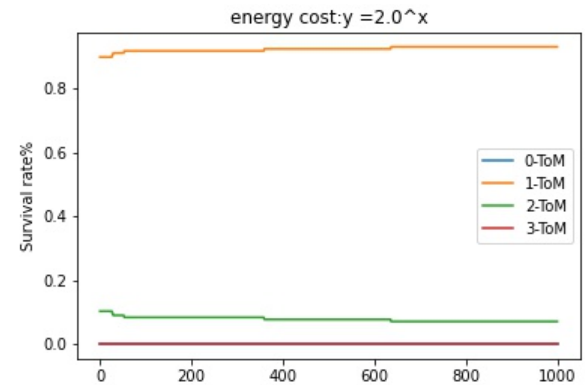


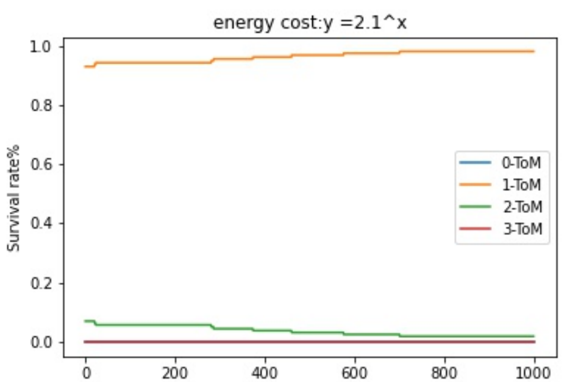




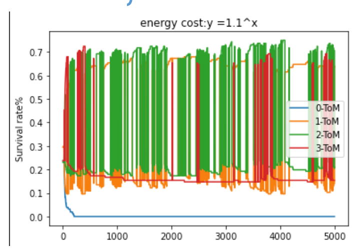






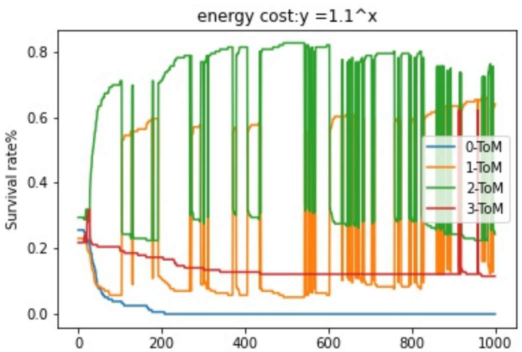


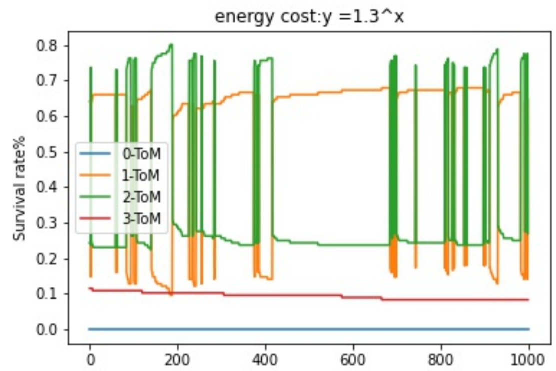
ToM-1 wins again! But it seems that in Y (energy cost%) = 1.1^{x} the system was not stable, even when I added the episode into 5,000

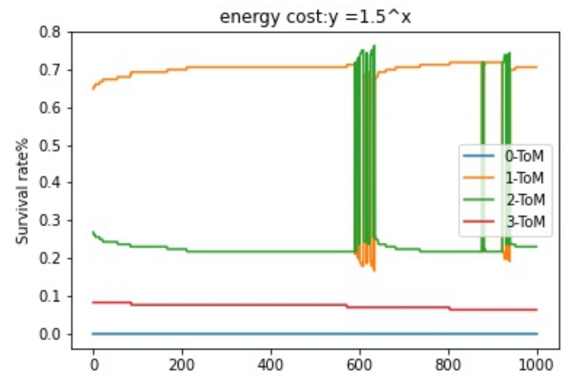


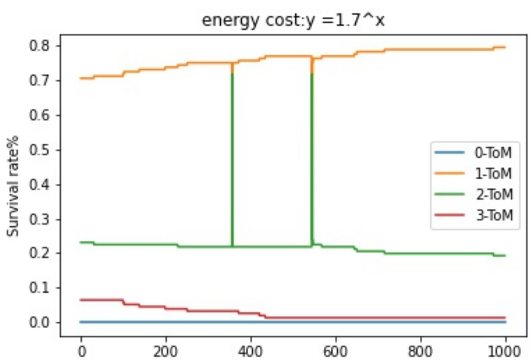
ToM-2 is social hub

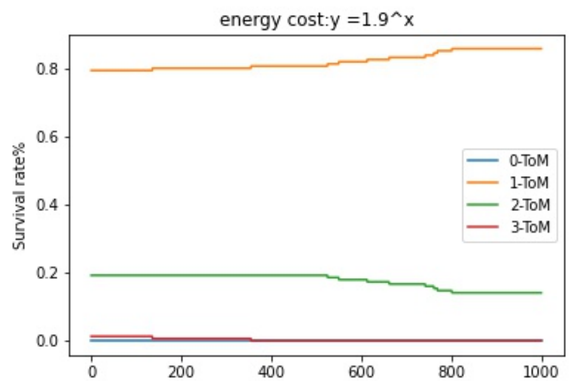
Results for social network

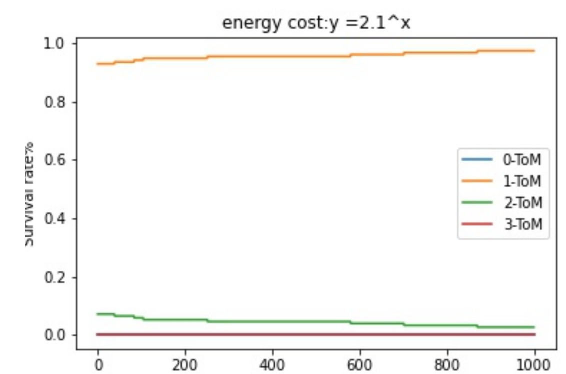


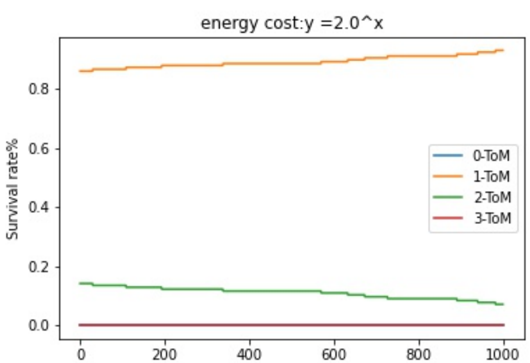




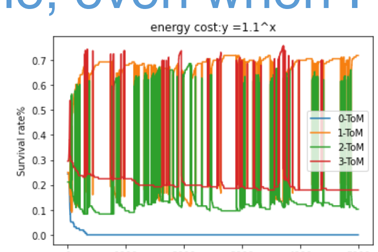






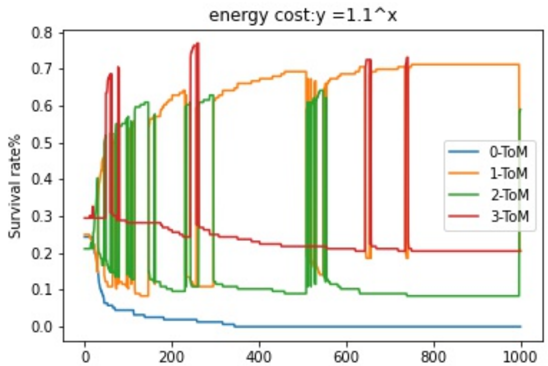


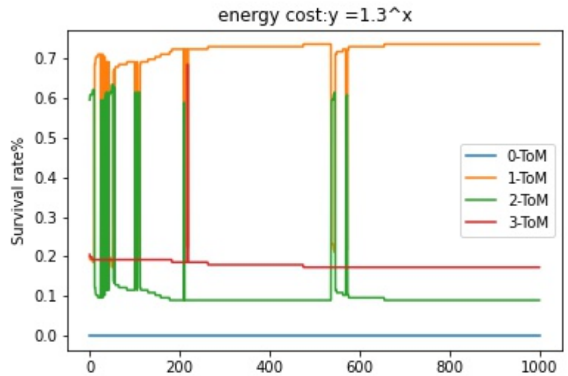
ToM-1 wins again! But it seems that in Y (energy cost%) = 1.1^x the system was not stable, even when I added the episode into 5,000

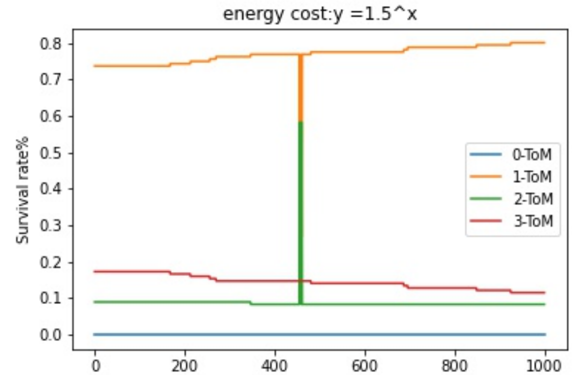


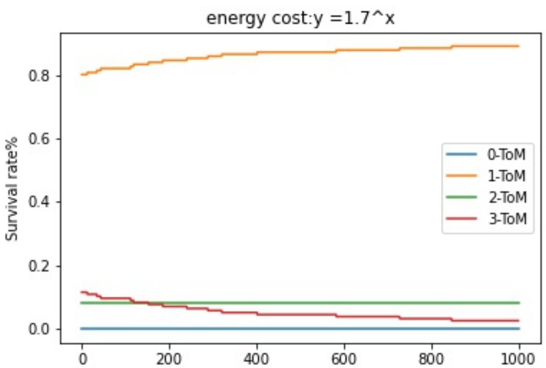
ToM-3 is social hub

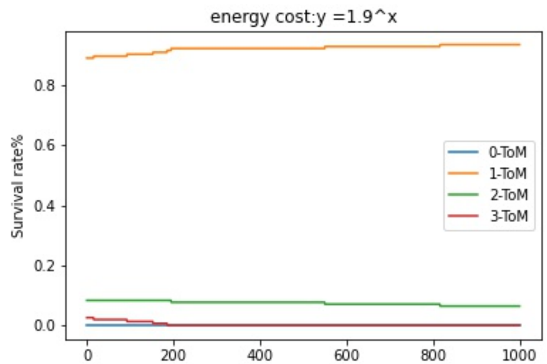
Results for social network

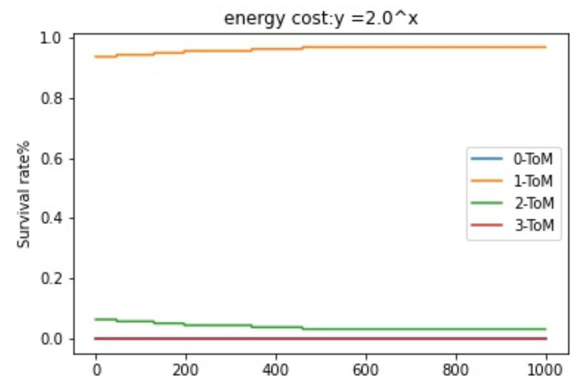


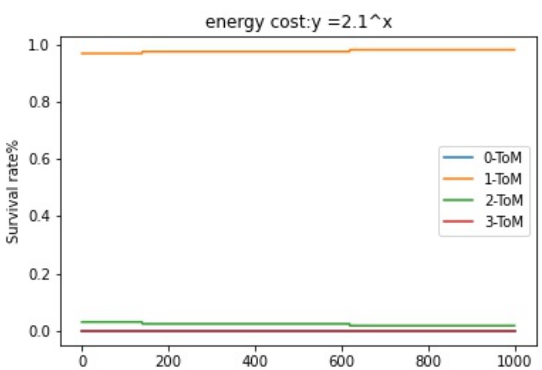






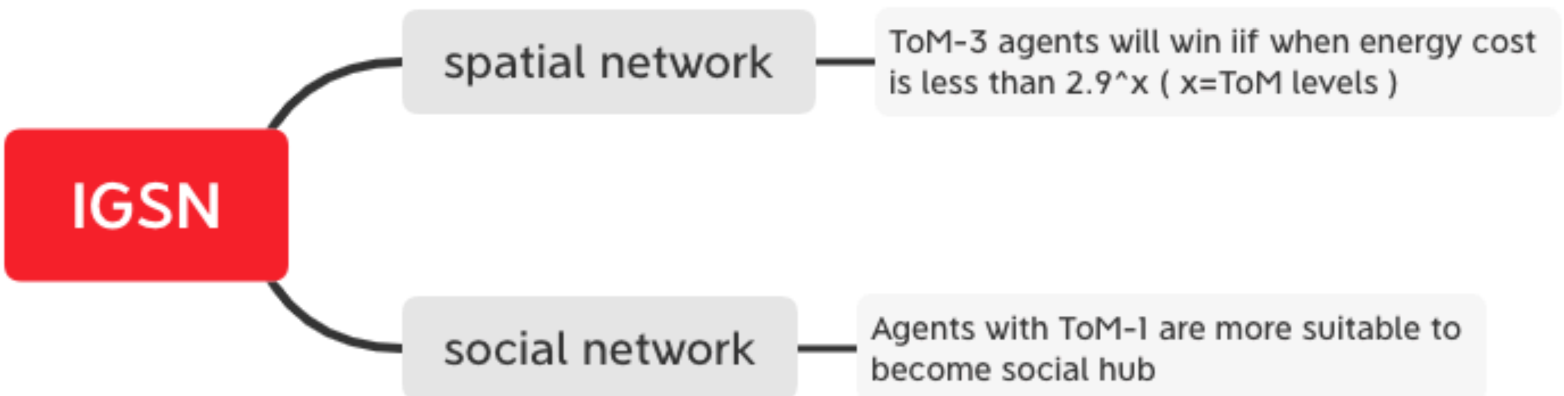






ToM-1 wins again!

Graphic conclusions



Code available

All codes (implemented in python) are available at:

https://github.com/psywalkeryanxy/interactive_gaming_social_network